UK Junior Mathematical Olympiad 2010

Organised by The United Kingdom Mathematics Trust

Tuesday 15th June 2010

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.

2. **The use of calculators, measuring instruments and squared paper is forbidden.**

3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).

4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

    For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

    Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

    Do not hand in rough work.

5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.

6. Questions B1-B6 are longer questions requiring *full written solutions*.
    This means that each answer must be accompanied by clear explanations and proofs.
    Work in rough first, then set out your final solution with clear explanations of each step.

7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.

8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols such as π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

A1 What is the value of $\dfrac{1}{1} + \dfrac{2}{\frac{1}{2}} + \dfrac{3}{\frac{1}{3}} + \dfrac{4}{\frac{1}{4}} + \dfrac{5}{\frac{1}{5}}$?

A2 Given that $x : y = 1 : 2$ and $y : z = 3 : 4$, what is $x : z$?

A3 Tom correctly works out 20^{10} and writes down his answer in full.
How many digits does he write down in his full answer?

A4 Three monkeys Barry, Harry and Larry met for tea in their favourite café, taking off their hats as they arrived. When they left, they each put on one of the hats at random. What is the probability that none of them left wearing the same hat as when they arrived?

A5 The sum of two positive integers is 97 and their difference is 37. What is their product?

A6 In the diagram, the equilateral triangle is divided into two identical equilateral triangles S and T, and two parallelograms Q and R which are mirror images of each other.
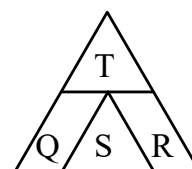
What is the ratio of area R : area T ?

A7 What is the largest possible angle in an isosceles triangle, in which the difference between the largest and smallest angles is 6°?

A8 The four square tiles having the designs as shown can be arranged to create a closed loop.
How many distinct closed loops, including the one shown here, can be made from the tiles?

(The tiles may be rotated, but a rotation of a loop is not considered distinct. A loop need not use all four tiles and may not use more than one of each type).
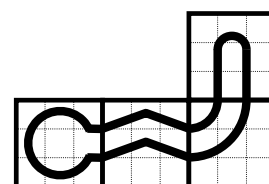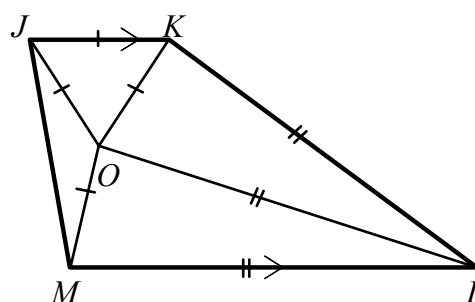
A9 Abbie, Betty and Clara write names on bookmarks sold for charity.

Abbie writes 7 names in 6 minutes, Betty writes 18 names in 10 minutes and Clara writes 23 names in 15 minutes.

If all of the girls work together at these rates, how long will it take them to write 540 names?

A10 In the diagram, JK and ML are parallel,
$JK = KO = OJ = OM$ and
$LM = LO = LK$.
Find the size of angle JMO.

Section B

Your solutions to Section B will have a major effect on your JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 In a sequence of six numbers, every term after the second term is the sum of the previous two terms. Also, the last term is four times the first term, and the sum of all six terms is 13.

What is the first term?

B2 The eight-digit number "*ppppqqqq*", where p and q are digits, is a multiple of 45.

What are the possible values of p?

B3 Jack and Jill went up a hill. They started at the same time, but Jack arrived at the top one-and-a-half hours before Jill. On the way down, Jill calculated that, if she had walked 50% faster and Jack had walked 50% slower, then they would have arrived at the top of the hill at the same time.

How long did Jill actually take to walk up to the top of the hill?

B4 The solution to each clue of this crossnumber is a two-digit number, not beginning with zero.

In how many different ways can the crossnumber be completed correctly?

Clues
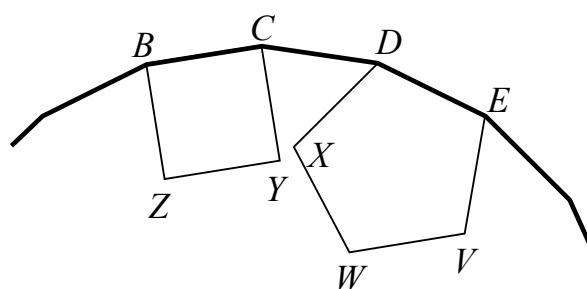Across
1. A triangular number
3. A triangular number

Down
1. A square number
2. A multiple of 5

B5 The diagram shows part of a regular 20-sided polygon (an icosagon) *ABCDEF*…, a square *BCYZ* and a regular pentagon *DEVWX*.

Show that the vertex *X* lies on the line *DY*.

B6 Sam has put sweets in five jars in such a way that no jar is empty and no two jars contain the same number of sweets. Also, any three jars contain more sweets in total than the total of the remaining two jars.

What is the smallest possible number of sweets altogether in the five jars?